Simulation of Amazon stock prices sing random walk theory

**OPR 9730 Project**

Harini Mittal

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Professor Arie Harel

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1. **Introduction**

Using random walk theory, this project simulates Amazon Inc’s stock prices for four years from May 6th, 2018 to May 6th, 2022 in oder to predict the range of stock prices on May 6th 2022 four years in advance. R software package is used for the simulation exercise. The simulated prices are compared to actual prices to draw conclusions based on random walk theory.

This paper is organized in five sections. Following introduction, section two explains random walk theory with an example. Section three gives the assumptions of random walk theory in stock markets. Section four lays down the methodology used for the study. Section five discusses the results of the analysis. Section six concludes the study with observations. The R code used for the exercise is in the appendix.

1. **What is random walk theory and how does it apply to stock prices?**

Random walk is a stochastic process which is used to describe a path taken by an object which is seemingly random, or unpredictable[[1]](#footnote-1). Think of a drunken man who has no control over his walk. His walk is not straight and can go anyway or movement of a molecule in a liquid. These movements are examples of random walk.

Using random walk theory, given enough trials, we are trying to find patterns in a seemingly random process. This helps us study a process and if possible, predict the outcome too. In this study, we use this random walk process to predict or simulate future stock prices.

Let us look at the example of a hypothetical company ABC Inc. with a stock price on 10th February. Let us suppose that there is a 50% probability that the prices will move up by $10 or move down by $10 the next day, Feb 11th. Therefore, the price of ABC Inc. can either be $90 or $110 the next day. Similarly, on Feb 12th, the prices can move up or down by $10 with a 50% probability. The $90 price on 11th Feb can go up to $100 or down to $80 on 12th Feb; the $110 price on 11th Feb can go up to $120 or down to $110 on 12th Feb and so on. The following table shows what the stock prices of ABC Inc will look like on 18th Feb following such a random process. Please note that we assume equal probability. The prices can take any path. A particular path of movement that the prices take is called the drift it takes.

***Table showing stock price simulation using random process for ABC Inc.***

Table

Description automatically generated with medium confidence

Source: <https://blog.quantinsti.com/random-walk/> accessed on May 12, 2022

1. **Assumptions of random walk theory in stock markets**

The following assumption are made when stock prices are simulated using random walk theory[[2]](#footnote-2):

* Random walk theory suggests that changes in stock prices have the same distribution and are independent of each other (iid).
* This theory infers that the past movement or trend of a stock price or market cannot be used to predict its future movement.
* It believes it is impossible to outperform the market without assuming additional risk.
* It considers technical analysis undependable because it results in chartists only buying or selling a security after a move has occurred.
* This theory considers fundamental analysis undependable due to the often-poor quality of information collected and its ability to be misinterpreted.
* This theory claims that investment advisors add little or no value to an investor’s portfolio. In his celebrated book ("A Random Walk Down Wall Street“, Burton Malkiel theorized that "a blindfolded monkey throwing darts at a newspaper's financial pages could select a portfolio that would do just as well as one carefully selected by experts." He in fact conducted a contest asking staff of wall street journal to throw darts at stock prices and compared it with the conclusions of experts for 140 stocks. He found that in 87 cases the expert won the contest, while the wall street staff that play the monkey dart-throwing process won in 55 cases. However, the experts were only able to beat the Dow Jones Industrial Average only 76%. Therefore, he concluded that at the most only in 50% cases, the experts prediction comes true.
* Sharp and Malkiel concluded that, due to the short-term randomness of returns, investors would be better off investing in a passively managed, well-diversified fund.
* Their conclusion is rooted in efficient markets hypothesis. The efficient market hypothesis states that stock prices fully reflect all available information and expectations, so current prices are the best approximation of a company’s intrinsic value.
* This would preclude anyone from exploiting mispriced stocks consistently because price movements are mostly random and driven by unforeseen events.

1. **Random walk analysis of Amazon Stock Prices[[3]](#footnote-3)**

This section elaborates the application of random walk theory to analyze Amazon’s stock prices. The study follows the following steps in its analysis.

Step 1: Obtain the log returns of Amazon’s stock beginning 2008-05-01 to 2018-05-01 using R code.

Step 2: Calculate the mean (mu) and standard deviation (sigma) of the log returns.

Step 3: To simulate the prices for next 4 years, begin all the simulations from the price on 05-01-2018.

Step 4:  Simulate the next days’ close price as per the Geometric Brown Motion formula. Text

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Where St is the stock price at time t, St-1 is the stock price at time t-1, μ is the mean daily returns, σ is the mean daily volatility t is the time interval of the step Wt is random normal noise. Generate the prices using the log-returns data using exponential growth rate to predict how much the stock will grow per day. The growth rate is randomly generated and dependent on the input values of mu and sigma.

Step 5: Compare the results with actual price data from Yahoo Finance for 05-02-2018 to 05-02-2022.

Step 6: Find the range of simulated prices on 05-02-2022.

Step 7: Draw conclusions.

1. **Discussion of the results**

Figure one shows the simulated prices for ~1000 trading days, approximately four years, using random walk using R. Figure two shows the actual prices for the four year prices of Amazon for the same period from Yahoo Finance.

As can be seen from the figures, there is an upward drift in both figures. But the trajectories are different.

***Figure 1***

Chart

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***Figure 2: Actual prices of Amazon from May 2nd 2018 to May 2nd 2022 obtained***

***from Yahoo Finance.***

***Chart

Description automatically generated***

Since the code did not include a set.seed(), to come up with a stable prediction, the price simulation needs to use the monte carlo method, where the prices are simulated repeatedly for accuracy. Figure 3 shows the Monte carlo simulation of

***Figure 3:Amazon Stock repeated 500 times.*** Chart

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A percentile check of the last day of the simulated prices predict that (AMZN)’s stock may reach the price of $9575.22 in four years (May 2022) time or crash to a $743.87 low.

***Percentile check of Amazon (AMZN)’s price after four years is***

## 0.5% 2.5% 25% 50% 75% 97.5% 99.5%   
## 743.8662 944.2857 1886.0158 2722.9865 4013.8743 6932.3167 9575.2241

If we compare these simulated percentile pricess with actual prices in May 2022 obtained from Yahoo finance, we can see that the closing price on May 2nd 2022 was $2490.00 (figure 4), within the 99% confidence interval.

***Figure 4: Actual prices in May 2022 (yahoo finance)***Timeline

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1. **Conclusions:**

Though the price was off the highest price simulated, these simulations are useful when one is interested in finding the Value at Risk (VaR) or the expected shortfall for a particular stock with a certain degree of confidence. According to Philippe Jorion, “VaR measures the worst expected loss over a given horizon under normal market conditions at a given level of confidence”. To perform such risk management analysis, one should generate as many simulations as possible as we did in this study.

**Appendix: R code used for the analysis**

library(quantmod)

library(xts)  
library(rvest)

library(tidyverse)

library(stringr)  
library(forcats)  
library(lubridate)

library(plotly)

library(corrplot)

library(dplyr)  
#Loads the company stock using ticker  
  
getSymbols("AMZN",from="2008-05-01",to="2018-05-01")

## [1] "AMZN"

#Stock returns in log  
  
AMZN\_log\_returns<-AMZN%>%Ad()%>%dailyReturn(type='log')  
#Mean of log stock returns   
  
AMZN\_mean\_log<-mean(AMZN\_log\_returns)  
#round it to 4 decimal places  
  
mean\_log<-c(AMZN\_mean\_log)  
  
mean\_log<-round(mean\_log,4)  
  
#standard deviation of log stock returns  
  
AMZN\_sd\_log<-sd(AMZN\_log\_returns)  
  
#round it to 4 decimal places   
  
sd\_log<-c(AMZN\_sd\_log)  
sd\_log<-round(sd\_log,4)  
  
  
#create data frame  
  
  
graphic1<-data.frame(rbind(c("AMZN",AMZN\_mean\_log,AMZN\_sd\_log)),stringsAsFactors = FALSE)  
  
  
  
  
graphic1<-data.frame(mean\_log,sd\_log)  
rownames(graphic1)<-c("AMZN")  
colnames(graphic1)<-c("Mean\_Log\_Return", "Sd\_Log\_Return")  
  
  
#Data frame contains the Amazon stock prices with each its average log return and standard deviation.  
#random walk: Rooted in past performance is not an indicator of future results. Price fluctuations can not be predicted with accuracy  
  
  
mu<-AMZN\_mean\_log  
sig<-AMZN\_sd\_log  
testsim<-rep(NA,1000)  
  
#generate random daily exponent increase rate using AMZN's mean and sd log returns  
  
#one year 252 trading days, simulate for 4 years   
# 4\*252 trading days  
  
price<-rep(NA,252\*4)  
  
#most recent price  
price[1]<-as.numeric(AMZN$AMZN.Adjusted[length(AMZN$AMZN.Adjusted),])  
  
#start simulating prices  
  
for(i in 2:length(testsim)){  
 price[i]<-price[i-1]\*exp(rnorm(1,mu,sig))  
}  
  
random\_data<-cbind(price,1:(252\*4))  
colnames(random\_data)<-c("Price","Day")  
random\_data<-as.data.frame(random\_data)  
  
random\_data%>%ggplot(aes(Day,Price))+geom\_line()+labs(title="Amazon (AMZN) price simulation for 4 years")+theme\_bw()

## Warning: Removed 8 row(s) containing missing values (geom\_path).

#monte carlo simulation: incredibly useful forecasting tool to predict outcomes of events with many random variables  
  
  
N<-500  
mc\_matrix<-matrix(nrow=252\*4,ncol=N)  
mc\_matrix[1,1]<-as.numeric(AMZN$AMZN.Adjusted[length(AMZN$AMZN.Adjusted),])  
  
for(j in 1:ncol(mc\_matrix)){  
 mc\_matrix[1,j]<-as.numeric(AMZN$AMZN.Adjusted[length(AMZN$AMZN.Adjusted),])  
 for(i in 2:nrow(mc\_matrix)){  
 mc\_matrix[i,j]<-mc\_matrix[i-1,j]\*exp(rnorm(1,mu,sig))  
 }  
}  
  
name<-str\_c("Sim ",seq(1,500))  
name<-c("Day",name)  
  
final\_mat<-cbind(1:(252\*4),mc\_matrix)  
final\_mat<-as.tibble(final\_mat)

colnames(final\_mat)<-name  
  
dim(final\_mat) #1008 501

## [1] 1008 501

final\_mat%>%gather("Simulation","Price",2:501)%>%ggplot(aes(x=Day,y=Price,Group=Simulation))+geom\_line(alpha=0.2)+labs(title="Amazon Stock (AMZN): 500 Monte Carlo Simulations for 4 Years")+theme\_bw()

#is it likely? Let us check the confidence intervals  
#Average stock daily return  
  
  
probs<-c(0.005,0.025,0.25,0.5,0.75,0.975,0.995)  
final\_mat[500,-1]%>%as.numeric()%>%quantile(probs=probs)

## 0.5% 2.5% 25% 50% 75% 97.5% 99.5%   
## 743.8662 944.2857 1886.0158 2722.9865 4013.8743 6932.3167 9575.2241

#Let us now see the actual returns  
  
#Loads the company stock using ticker  
  
getSymbols("AMZN",from="2018-05-02",to="2022-05-02")

## [1] "AMZN"

AMZN%>%Ad()%>%chartSeries()

1. <https://blog.quantinsti.com/random-walk/> accessed on May 12, 2022 [↑](#footnote-ref-1)
2. [https://www.investopedia.com/terms/r/randomwalktheory.asp](https://towardsdatascience.com/analyzing-stocks-using-r-550be7f5f20d) accessed on May 12, 2022 [↑](#footnote-ref-2)
3. <https://blog.quantinsti.com/random-walk/> accessed on May 12, 2022 [↑](#footnote-ref-3)